

# **Radius of Curvature Measurement Plan for AMSD**

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## The Radius of Curvature (RoC) Problem

- Definition of RoC for AMSD
- Why precision RoC measurement can be difficult
- What can be done to overcome the difficulty

## The RoC Solution

- Use interferometer data along with ADM ranging
- Large number of data points with high precision

## RoC Measurement

- Absolute surface measurement
- Curve fit to paraboloid

## Error

## An Alternative



## Requirements for AMSD

- **AMSD parent paraboloid has 10 m vertex RoC.**
- **AMSD segment center 1400 mm off-axis with flat-to-flat diameter of 1200 mm.**
- **SOW requires parent vertex RoC = 10 m  $\pm$  1 mm.**
- **MSFC needs capability to verify compliance of AMSDs at cryo temperatures.**
- **Means capability to measure vertex RoC for a segment with no physical vertex using no vertex fixtures with accuracy better than 1 mm.**



**There is not a physical vertex**

- **Location of vertex cannot be measured directly.**
- **A standard definition of RoC must be established.**

**Ways to define vertex RoC**

- **Must be defined as the vertex RoC of some best-fit surface**
- **Best-fit surface can be a paraboloid, or a general conic.**
- **We define vertex RoC as that of the best-fit paraboloid, since the nominal surface is paraboloidal and large errors in the conic are not expected (would cause WFE requirement violation).**

# Why Precision Can Be Difficult



- Remote determination of RoC requires, in general, a surface fit of a set of surface sag measurements.
- Error is propagated through fit and depends ultimately on dimensions of test arrangement: the smaller the pupil is relative to the RoC, the worse the precision in the determination of the vertex RoC.
- For scale of AMSD (where segment decenter is about 0.14 times vertex RoC & pupil diameter is about equal to decenter) and for small number ( $\sim 10$ ) of sag data points, precision in RoC determination is about 50-100 times worse than precision in individual sag measurements.
- Similar problems plague other indirect RoC measurement techniques.



# What We Have to Do to Improve Precision

**Two ways to improve precision  
in measurement of vertex RoC:**

- 1. Acquire a large number of data points.**
- 2. Make individual sag measurements with better accuracy.**



## Interferometric Measurement of Sags Combined with Precision Part Location

**Knowledge about reference wavefront  
is required to calculate absolute surface.**

- **Absolute surface determined by adding figure error (in terms of sag) to reference wave sag (as produced by null).**
- **Reference wavefront known by measuring null-AMSD separation using ADM (accuracy of 20 microns or less).**
- **Relies on either strict alignment of test article (baseline approach) or separation of misalignment from misfigure after the fact.**
- **Yields vertex RoC by fitting of absolute surface to best paraboloid.**

# Interferometer Yields Large Data Set



**An extremely large set of sag data is already produced by interferometer – task is in accurately knowing shape of surface to which sags are referenced.**

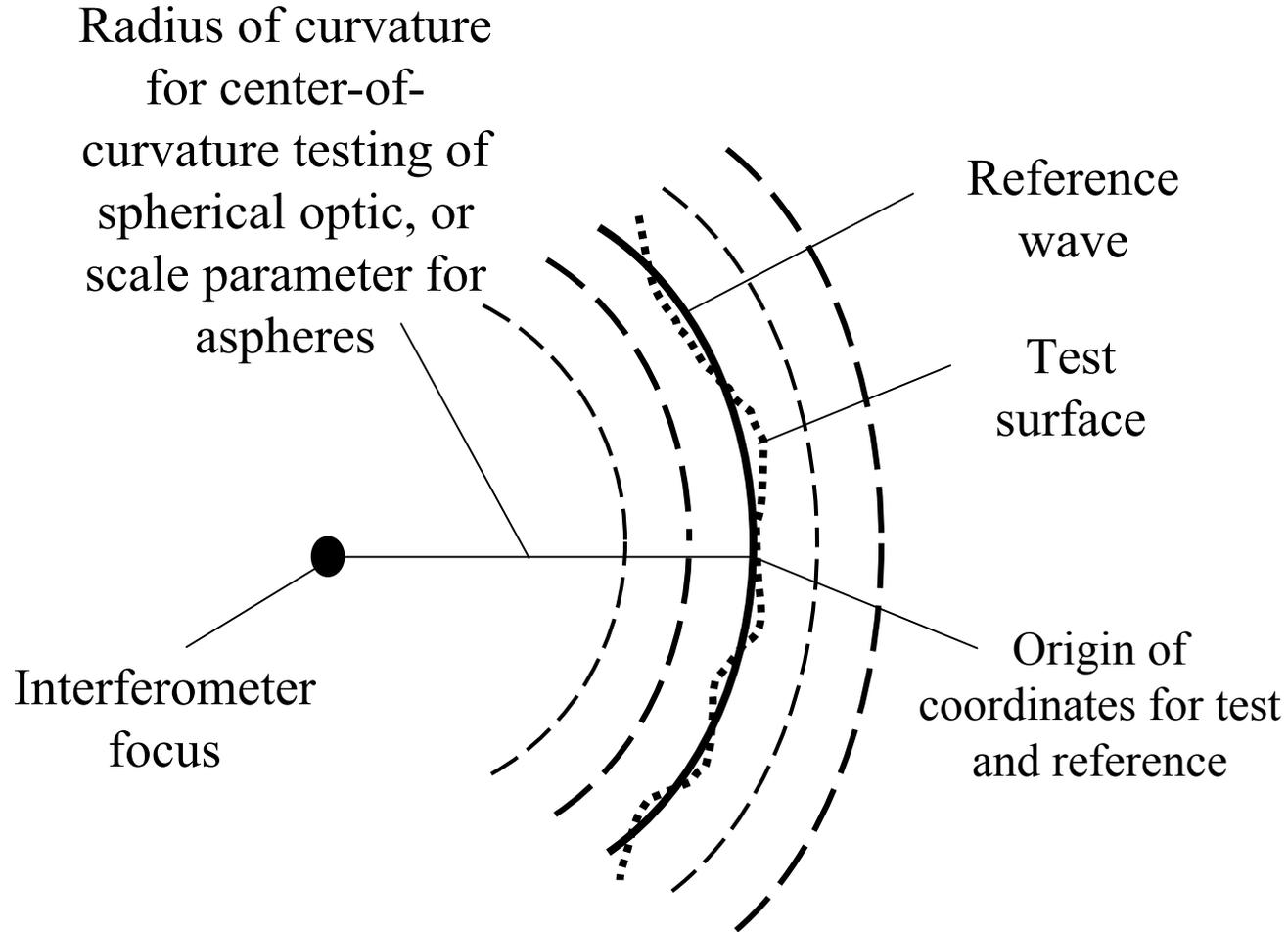
- Analysis shows that RoC measurement precision improves with reciprocal of square root of number of sag data points, much like standard deviation of mean of a set of measurements.**
- In case of AMSD, will have about  $5 \times 10^5$  sag data points as output from interferometer.**
- Mitigates error introduced into curve fitting algorithm – in other words, curve-fitting of a small set of sag data measurements (10-20) made with ADM not feasible.**

# Interferometer Provides Good Sag Measurement Accuracy

**Interferometer provides means of measuring sags with sub-wavelength accuracy.**

- **Interferometry is inherently precise. This is why it is used to measure surface figure error in first place.**
- **Accurate determination of absolute surface using interferometric wave front error requires mitigation of other error sources, such as uncertainty in location of reference wave, pixel error, null error, and environmental errors such as vibration & turbulence.**
- **Ultimately, error introduced by interferometry is not limiting error source.**

# Ascertaining Reference Wave

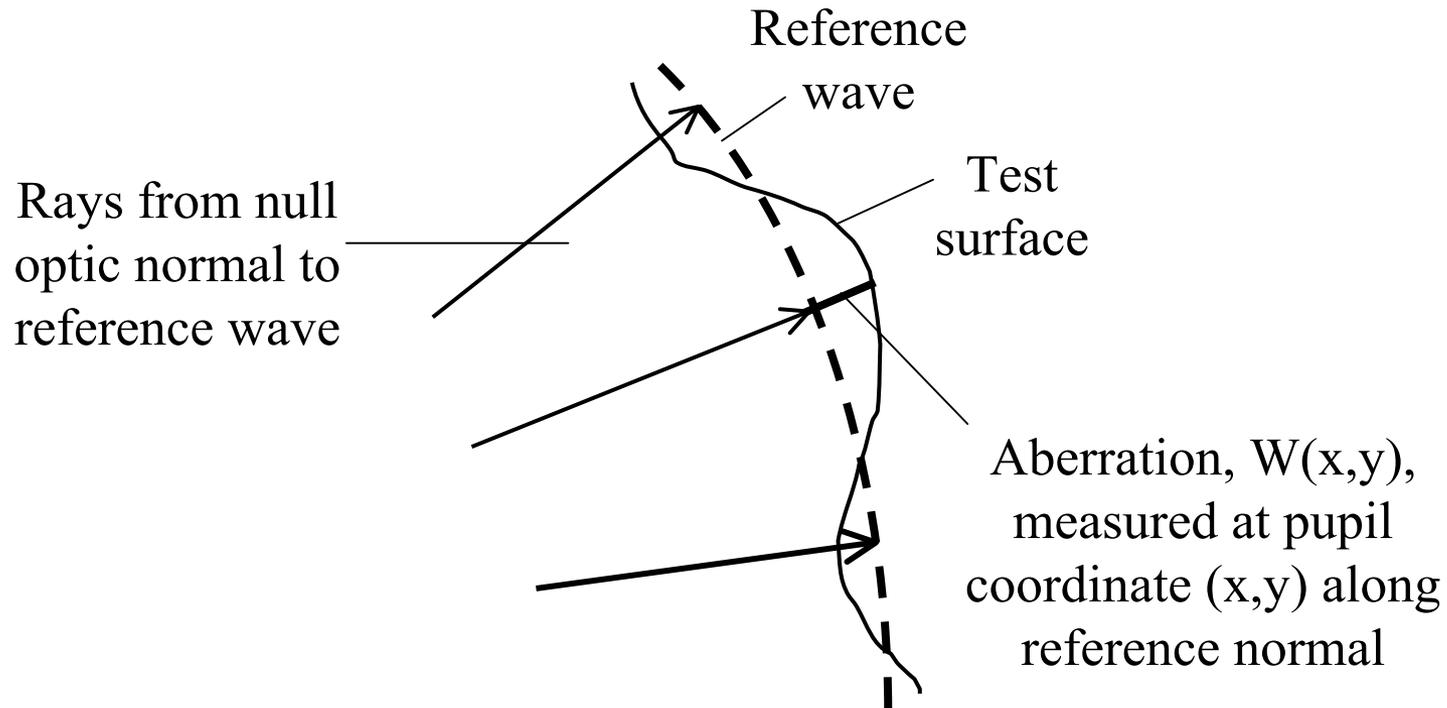


# Absolute Surface Determination

**Absolute surface obtained from wavefront measurement & segment position measurement.**

- **Figure error measured by interferometer as detailed previously (by Patrick Reardon). Measured wavefront (interferometer output) transformed to figure error relative to nominal paraboloid after measurement of part position.**
- **Figure error transformed into pure sag error in parent coordinates.**
- **Sag error added to true reference wavefront to yield absolute surface. This surface is represented by a discrete set of data.**

# Wavefront Aberration in Null Configuration Is Not Sag Difference



# Curve Fit to Paraboloid

**Data set describing absolute surface is fit to a paraboloid.**

- **Curve fit directly yields value for parameter  $1/(2R)$ , reciprocal of twice vertex RoC, by minimizing sum of squares of error between absolute surface data and a perfect parabola, with  $R$  being the optimization variable.**
- **Various software available for this fitting – believe Code V optimization routine would be ideal. Absolute surface input to Code V using “interferometer” file and  $R$  varied to optimize surface figure with  $K$  set to  $-1$ .**
- **Since a paraboloid is axially symmetric, fit could be done in 2-D only (curve fit, not surface fit), making computation easier.**

# Summary of RoC Measurement Procedure



- **AMSD RoC measurement begins with figure measurement using interferometer (difference between mirror surface & reference wavefront).**
- **Reference wavefront found by measuring longitudinal position of segment with respect to CGH null (or with respect to location of perfect  $R = 10$  m paraboloidal reference wavefront). Reference wavefront is then analytically propagated to measured longitudinal position. This measurement is performed with high accuracy using absolute distance meter.**
- **Output of interferometer, along with knowledge of reference wavefront, is used to calculate absolute surface of mirror.**
- **Paraboloid is fit to absolute surface to yield vertex RoC. Note that general conic (RoC & K) could also be fit to absolute mirror surface - still investigating whether this is needed.**

# RoC Error Sources

**Sources of error in determination of RoC using this method are essentially same as those for figure measurement.**

- **Data used to determine absolute surface are obtained from measurement of figure.**
- **So, method uses only data already acquired during figure measurement with same errors.**
- **Propagation of error proceeds through least-squares fit, however, where errors tend to be amplified.**



Formula for RoC, obtained through least-squares fit, can be written as follows

$$RoC = \left[ 2 \sum_i z_i \frac{Nr_i^2 - \sum_k r_k^2}{\left( N \sum_k r_k^2 \right) - \left( \sum_k r_k \right)^2} \right]^{-1}$$

Instead of full analytic propagation of errors, errors simulated as Gaussian random noise.

- Error in surface sag is about 30 nm RMS. This represents a rollup of all contributing error sources, including 20 micron ADM-induced error in longitudinal location of segment with respect to reference wave.
- According to conservative estimates, the above produces an RMS error in vertex RoC of less than 40  $\mu\text{m}$ .

## An Alternative Method

An alternative method for measuring RoC is to measure sagittal & tangential radii of curvature on segment (can be done optically in a variety of ways). This method can yield either the vertex RoC of the best-fit paraboloid or the vertex RoC of the best-fit conic.

$$RoC = \Delta\rho_{TS} \frac{\cos \theta_N}{\tan^2 \theta_N}$$

**RoC of best-fit  
paraboloid.**

$$RoC = \sqrt{\frac{R_S^3}{R_T^2}}$$

**RoC of best-fit  
conicoid.**

## RoC Summary

**Have developed method of measuring RoC of an OAP to a currently-estimated accuracy of about 40 microns, that does not require auxiliary hardware at either the mirror or at the center of curvature.**

**Method is based on approaches developed and used on large, off-axis, conic mirrors over the past twenty years to separate misalignment from actual misfigure.**

**Nearly same method independently developed and verified by Optical Research Associates – leads to high confidence.**

**Have a second approach that will also meet the AMSD requirement for absolute RoC – would require some straight-forward auxiliary optics.**